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Anomalous magnetic moment of ^{9}C and shell quenching in exotic nuclei

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Abstract. We discuss a mechanism of the anomalous spin expectation value of the ^{9}C-^{9}Li mirror pair in relation to the shell structure of exotic nuclei. In a similar way to the $N = 20$ shell gap for neutron-rich nuclei, the $N = 8$ shell gap should be rather narrow toward smaller Z from an empirical determination of the shell gap. The resulting ground state of ^{9}Li is still dominated by the normal configurations, whereas that of the mirror nucleus ^{9}C can be mixed with the intruder configurations triggered by the Thomas-Ehrman effect. This asymmetry of the ground state accounts for the experimental anomaly.

PACS. 21.10.Ky Electromagnetic moments – 21.60.Cs Shell model – 27.20.+n $6 \leq A \leq 19$

1 Introduction

The magnetic moment carries much information on the distribution of the total angular momentum among spin and orbital parts of protons and neutrons, since the g factor of each part is quite different from one another. Thus, the magnetic moment is of great help in understanding the single-particle structure, deformation, configuration mixing, etc. From magnetic moments of mirror nuclei, one can directly deduce the isoscalar spin expectation value [1] as

$$\langle \sigma_z \rangle = \frac{\mu(T_z = +T) + \mu(T_z = -T) - J}{0.38}, \qquad (1)$$

where the mirror symmetry between the pair is assumed. In general, the absolute value of this spin expectation value for odd nuclei does not exceed a single-particle estimate [1] because of the strong pairing correlation. The configuration mixing decreases the absolute value of the spin expectation value due to the mixing with the spin-orbit partner. However, it has turned out that the spin expectation value of the ^{9}C-^{9}Li pair has an extraordinary large value, 1.44, from the magnetic moment of ^{9}C measured for the first time by Matsuta *et al.* [2].

In the present paper, we discuss the shell structure varying from stable to unstable nuclei, and point out that it can lead to this anomalous spin expectation value. In the next section, we first survey results of previous theoretical studies on this problem, and indicate the need for inclusion of something not explored in depth so far.

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2 Knowledge from previous calculations

The shell model with an appropriate effective interaction successfully describes low-lying structure of nuclei in a systematic way. Good examples are found in the p-shell calculation by the Cohen-Kurath interaction [3], the sd-shell calculation by the USD [4], and the pf-shell calculations by the KB3 [5] and the GXPF1 [6]. These shell-model calculations give good magnetic moments, too. The spin expectation value of the ^{9}C-^{9}Li pair is calculated by available realistic p-shell model interactions, all of which give a value close to the single-particle estimate, 1, almost independently of the effective interaction.

We now mention the nucleon g factors adopted in the calculation of the magnetic moment. Brown and Wildenthal [7] examined empirically optimum g factors and their effect on the magnetic moment within the full sd-shell framework. It was found in [7] that the free nucleon g factors give a good magnetic moment as a whole, but a certain improvement of the isoscalar spin expectation value is attained with effective g factors. Namely, the absolute value of the spin expectation value by the free nucleon g factors is somewhat larger than the experimental value typically by ~ 0.1. This probably means that the effective g factors include renormalization of the second-order configuration mixing, whose effect is less than the first-order one but does work to reduce the spin expectation value. Namely, the use of the effective g factors does not explain the anomalous value for the ^{9}C-^{9}Li pair.

In the shell-model, the mirror symmetry between the pair nuclei is assumed by using the isospin-conserving interaction. In ref. [8], the spin expectation value was calculated including the isospin-nonconserving process. The value was improved by 0.09 in [8], but there still remains a certain deviation. In models describing the clusterization [9,10], the isospin symmetry is not assumed. The values by those models exceed the single-particle value by about 0.1, but are still rather short of the experimental one. This similar results between the shell model and the cluster models probably reflect the resemblance of the wave functions between those models. In order to go beyond the previous results, it may be essential to include configurations not only within the p shell but also involving excitations into the sd shell. In this situation, it is of importance to investigate the shell gap between the p to sd shells. In the next section, we summarize recent knowledge about the shell structure in exotic nuclei.

3 Shell evolution

Since much information on the structure of neutron-rich nuclei has been recently accumulated owing to experimental developments, the shell structure of unstable nuclei has emerged gradually. This context is well investigated in the $N \sim 20$ region in relation to the disappearance of the magic structure. In the so-called "island of inversion" picture [11], the disappearance of the magic structure occurs in some of $N = 20$ isotones, ^{30}Ne, ^{31}Na and ^{32}Mg, confirmed by several experiments. On the other hand, this picture predicts that the disappearance of the magic number does not occur in any $N < 20$ nuclei.

We shall now define the shell gap as the difference of the so-called effective-single-particle energies (ESPE) (see, *e.g.*, [12]) of the relevant orbits, including effects of the two-body interaction in the form of the monopole interaction. The $N = 20$ shell gap by the "island of inversion" picture is only weakly dependent on the proton and neutron numbers, and a somewhat constant ~ 6 MeV gap persists from stable to unstable nuclei.

Detailed structure of $N < 20$ nuclei has recently been accessible (see [13,14] for the magnetic moment). Experimental results [13,14] show that the boundary of the "island of inversion" must be extended from the original [11]. In relation to the shell structure, this extension indicates that the $N = 20$ shell gap should be narrower than that of stable nuclei. Indeed, a Monte Carlo shell-model study [15] shows that at $N = 19$ the intruder state is superior to the normal state with an interaction giving narrowing $N = 20$ shell gap toward smaller Z [12].

This varying shell gap has been discussed from a more general viewpoint of the effective interaction by Otsuka *et al.* [16] including the author of the present paper. Following this argument, referred to as the "shell evolution", the $N = 8$ shell gap should be changed as the proton number varies.

4 Effect of shell quenching on the anomalous spin expectation value

4.1 Empirically determined N = 8 shell gap

To calculate the ground state of ^{9}Li (*i.e.*, the mirror nucleus of ^{9}C), it is desirable to fix the $N = 8$ shell gap for this nucleus. For Li isotopes, however, direct experimental information to determine the gap is missing. On the other hand, the shell gap for $Z = 4$–6 can be empirically determined by adjusting positive- and negative-parity energy levels of $N = 7$ isotones, which can be made use of to fix the shell gap at $Z = 3$.

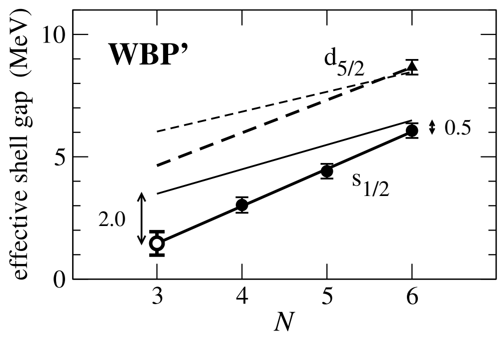

Fig. 1. $N = 8$ shell gap as a function of the proton number. The thin lines are those of the original WBP interaction, while the marks guided by thick lines indicate the empirically determined one. The open circle is the shell gap at $Z = 3$ extrapolated from the $Z = 4$–6 data.

In this study, we perform p-sd shell model calculations. As the effective interaction we start with the WBP interaction [17] giving a good description of the abnormal-parity states, mainly of near-stable nuclei, in this region. The original WBP interaction is designed to be used in the pure $0\hbar\omega$ and $1\hbar\omega$ model spaces, while in the present study (see ref. [18] in detail), the mixing with the higher excited configurations is included. The inclusion of the mixing plays an essential role particularly in the case that the normal and intruder states compete with each other. Since the mixing may have different effects between the $0\hbar\omega$ and $1\hbar\omega$ states, we should re-examine the shell gap suitable for this model space.

In fig. 1, we compare the shell gap by the original WBP interaction with the empirical shell gap reproducing levels of abnormal-parity states of the $N = 7$ isotones. The empirical shell gap is rather similar to the original for $Z = 6$, whereas the difference between them increases as Z decreases. Namely, the evolution of the $N = 8$ shell gap more transparently develops in the extended model space than that in the small space. The reason for the difference is given in the following. The $0\hbar\omega$ states couple mainly with the $2\hbar\omega$ ones, while the $1\hbar\omega$ states do with the $3\hbar\omega$ ones. Since in general the $n\hbar\omega$ states are rapidly located higher as n increases, the coupling with the $2\hbar\omega$ states in the normal parity is stronger than that with the $3\hbar\omega$ states in the abnormal parity. Thus, it is likely that the mixing favors the normal parity state, which in fact accounts for about half of the difference of the shell gap at $Z = 4$. The other half is accounted for by the difference of the treatment of the ^{4}He core: in the pure $0\hbar\omega$ and $1\hbar\omega$ calculation, the breaking of the core is allowed only for the $1\hbar\omega$ state. This means that only the negative-parity states have room for the gain of the correlation energy through the breaking of the ^{4}He core. On the other hand, in the present p-sd shell calculation the core breaking is not allowed for both parity states, equally.

As a result, the shell gap for $Z = 3$ extrapolated from the empirical ones of $Z = 4$–6 is much narrower than that of the original interaction as shown in fig. 1.

4.2 Phenomenological treatment of the Thomas-Ehrman effect

We move the ESPE of the sd orbits for $Z = 3$ to be the same as the extrapolated one in fig. 1, simply shifting the relevant bare single-particle energies. Even with this narrowing shell gap, the ground state of ^{9}Li calculated by the shell model is still dominated by the normal configurations. Consequently, the spin expectation value is calculated to be a normal one, also, under the assumption of the mirror symmetry for the ^{9}C-^{9}Li pair.

While ^{9}Li is a sufficiently bound nucleus, its mirror pair, ^{9}C, is a very loosely bound one having $S_p = 1.3$ MeV due to the repulsive Coulomb force. In this situation, single-particle orbits above the Fermi level can be lowered from those of the mirror nucleus owing to the Thomas-Ehrman effect. In this study, the Thomas-Ehrman effect is incorporated in a phenomenological way, *i.e.*, just by shifting the relevant single-particle energies in the shell-model calculation (see ref. [18] in more detail). In many cases, the Thomas-Ehrman effect emerges as the shift of energy levels involving the relevant orbit, whereas the component of the many-body wave function barely changes. In the present case, on the other hand, the ground state is gradually mixed with the intruder state once the Thomas-Ehrman effect is switched on. As a result, the magnetic moment of ^{9}C is shifted from that of the mirror wave function of ^{9}Li, and the spin expectation value calculated by eq. (1) becomes close to the experimental value with a reasonable shift of the single-particle energy. We stress that this softness of the ground-state configuration is most attributed to the narrow $N = 8$ shell gap as fixed in sect. 4.1.

It is worthwhile to discuss how the mixing with the intruder configurations accounts for the experimental spin expectation value. Table 1 compares the expectation values of the angular-momentum operators, l and s, in the ground state of ^{9}C between different calculations. For the $0\hbar\omega$, *i.e.*, p-shell calculation, the distribution of the total angular momentum, 3/2, is rather similar to the pure single-particle value. On the other hand, most of the orbital angular momentum, carried by neutrons in the $0\hbar\omega$ ground state, moves to valence protons in the $2\hbar\omega$ ground state. To be intuitive, since a large (prolate) deformation is favored in this intruder state as the Nilsson model predicts, it is very likely that the proton orbital angular momentum is large as a consequence of the collective rotation. It should be noted that once the mixing occurs by the lowering of the $0s_{1/2}$ orbit, the d orbits are involved, too, producing the present angular momentum distribution. In a well mixed wave function, the distribution is in between as shown table 1. Because the g-factor of the proton orbital angular momentum is positive, the magnetic moment is shifted positively, which pulls the calculated spin expectation value toward the experimental one.

Table 1. Expectation value of the angular-momentum operators for the ground state of ^{9}C, compared between the single-particle $\nu(0p_{3/2})^1$ state (SP), the $0\hbar\omega$ and $2\hbar\omega$ shell-model calculations without the mixing, and the $(0+2)\hbar\omega$ calculations. For the $(0+2)\hbar\omega$ calculations, the values with/without the Thomas-Ehrman (TE) effect are shown. Note that the experimental magnetic moment is $(-)1.3914(5)\mu_N$ [2] or $(-)1.396(3)\mu_N$ [8].

	SP	$0\hbar\omega$	$2\hbar\omega$	$(0+2)\hbar\omega$	
				w/o TE	w/ TE
$\langle l_z^{\mathrm{p}} \rangle$	0	0.14	0.83	0.24	0.44
$\langle l_z^{\mathrm{n}} \rangle$	1	0.84	0.15	0.75	0.56
$\langle s_z^{\mathrm{p}} \rangle$	0	0.02	0.02	0.02	0.01
$\langle s_z^{\mathrm{n}} \rangle$	0.5	0.50	0.50	0.49	0.49
$\langle \mu \rangle$	−1.91	−1.66	−0.95	−1.54	−1.38

5 Some other possible clues

At present, there is no direct experimental information indisputably indicating the large breaking of mirror symmetry in the ^{9}C-^{9}Li pair. In order to obtain clues which can be related to the breaking, the followings would give some hints, although they may be rather subtle effects to be treated delicately.

5.1 Asymmetry of the β decay between the mirror nuclei

From recent measurements of the β decay from the ^{9}Li and ^{9}C [19,20], there is a large difference in their $B(GT)$ values for the decays having the largest $B(GT)$ value. This indicates that the mirror symmetry must be broken for either (or both) the parent ground states or the daughter states of this pair.

5.2 Deviation from the IMME

It has been known that masses of the $T = 3/2$ isobaric quartet well follow the so-called the isobaric multiplet mass equation (IMME) [21]. For $A = 9$, however, the deviation from the IMME is larger than a theoretical estimate [22]. It is possible that this deviation is caused by a large energy gain due to the mixing with the intruder configurations in ^{9}C.

5.3 Energy levels

Only one excited state has been reported for ^{9}C. It is located at 2.22 MeV, whereas the known first excited state of ^{9}Li lies at 2.69 MeV [23]. The spin/parity of neither state has been measured yet. If they are the mirror levels involving the excitation of the last odd nucleon, *i.e.* $1/2^-$, as expected by the p-shell model, this energy difference appears much larger than that seen in typical ones. This difference may be a signature of the large mirror asymmetry.

6 Summary

We examined the shell structure of unstable nuclei in detail and discussed its effect on the anomalously large isoscalar spin expectation value known for the ^{9}C-^{9}Li mirror pair. Similarly to the variation of the $N = 20$ shell gap from stable to unstable nuclei, it turned out that the $N = 8$ shell gap should also become rather narrow toward smaller proton numbers, obtained from an empirical determination of the shell gap by a p-sd shell-model calculation. In the present shell-model calculation, we included the mixing between different $\hbar\omega$ configurations. With this narrow shell gap, although the normal configurations still dominate the ground state of ^{9}Li, this state is softly mixed with the intruder state, which would occur in ^{9}C due to the Thomas-Ehrman effect. Some other viewpoints were suggested, possibly related to the breaking of the mirror symmetry in the ^{9}C-^{9}Li pair.

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